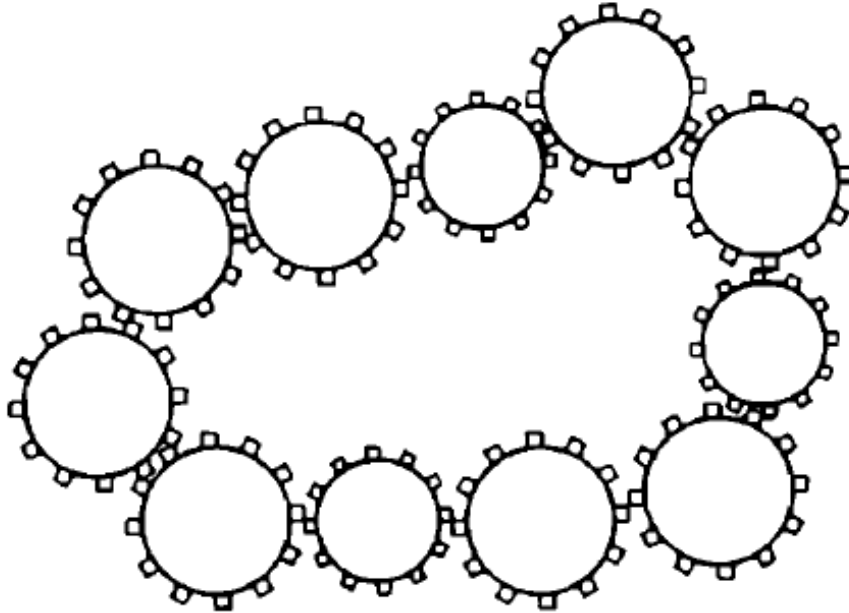


Έντεκα γρανάζια τοποθετούνται σε μια αλυσίδα όπως στο σχήμα που φαίνεται. Μπορούν όλα τα γρανάζια να περιστρέφονται ταυτόχρονα? Δικαιολογήστε την απάντησή σας.



Solution. The answer is no. Suppose that the first gear rotates clockwise. Then the second gear must rotate counter-clockwise, the third clockwise again, the fourth counter-clockwise, and so on. It is clear that the “odd” gears must rotate clockwise, while the “even” gears must rotate counter-clockwise. But then the first and eleventh gears must rotate in the same direction. This is a contradiction.

The main idea in the solution to this problem is that the gears rotating clockwise and counter-clockwise alternate. Finding objects that alternate is the basic idea in

Μπορούμε να ζωγραφίσουμε μια κλειστή πολυγωνική γραμμή με 9 ευθύγραμμα τμήματα καθένα από τα οποία τέμνει ακριβώς ένα από τα υπόλοιπα ευθύγραμμα τμήματα?

Solution. If such a closed path were possible, then all the line segments could be partitioned into pairs of intersecting segments. But then the number of segments would have to be even.

Let us single out the central point in this solution: if a set of objects can be partitioned into pairs, then there are evenly many of them. Here are some similar

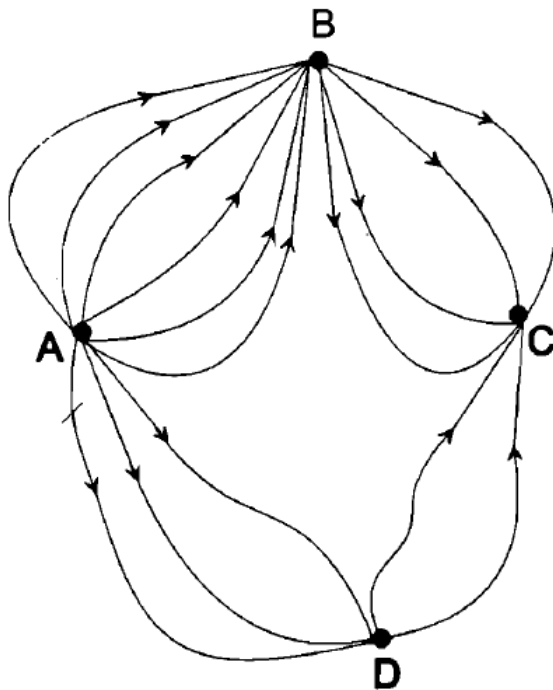
Υπάρχουν 5 διαφορετικά φλιτζάνια και 3 διαφορετικά πιατάκια σε ένα κατάστημα υαλικών. Πόσοι συνδιασμοί υπάρχουν αν θέλετε να αγοράσετε 1 φλιτζάνι και 1 πιατάκι?

Solution. First, let us choose a cup. Then, to complete the set, we can choose any of three saucers. Thus we have 3 different sets containing the chosen cup. Since there are five cups, we have 15 different sets ($15 = 5 \cdot 3$).

Problem 2. There are also four different teaspoons in the “Tea Party” store. How many ways are there to buy a set consisting of a cup, a saucer, and a spoon?

Solution. Let us start with any of the 15 sets from the previous problem. There are four different ways to complete it by choosing a spoon. Therefore, the number of all possible sets is 60 (since $60 = 15 \cdot 4 = 5 \cdot 3 \cdot 4$).

Στην Wonderland υπάρχουν οι πόλεις A,B,C και D. Στο παρακάτω σχήμα φαίνονται οι δρόμοι που ενώνουν τις πόλεις καθώς και οι κατευθύνσεις που έχουν. Με πόσους τρόπους μπορείτε να πάτε από την πόλη A στην πόλη C?



Solution. Consider two cases: our route passes either through B or through D. In each case it is quite easy to calculate the number of routes—if we drive through B then we have 24 ways to drive from A to C; otherwise we have 6 ways. To obtain the answer we must add up these two numbers. Thus we have 30 possible routes.

